Robust and Efficient Location of T-Wave Ends in Electrocardiogram

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Abstract

A new algorithm is proposed in this paper for T-wave end location in electrocardiogram (ECG). It mainly consists of the computation of an indicator related to the area covered by the T-wave curve and delimited in a special manner. Based on simple assumptions, essentially on the concavity of the T-wave form, it is proved that the maximum of the computed indicator inside each cardiac cycle coincides with the T-wave end. The algorithm is robust to measurement noise, to wave form morphological variations and to baseline wander. It is also computationally very simple: the main computation can be implemented as a simple finite impulse response (FIR) filter.

1. Introduction

Automatic processing of electrocardiogram (ECG) has been one of the earliest applications of modern digital computers. In this vast field, automatic detection of wave forms in ECG signals is still an active research topic, as demonstrated by recent publications [1, 2, 3, 4]. Because of the great morphological variations in ECG signals, it is difficult to design automatic and widely applicable algorithms. This difficulty partly explains the continuous efforts made by researchers on ECG signal processing.

It is widely acknowledged that T-wave end location is the most difficult one among wave form boundary location problems, due to the slow transition in the signal around each T-wave end, occasionally corrupted by noise. The purpose of this paper is to propose a new algorithm for the location of T-wave end. Its most remarkable difference from existing algorithms is its consistency proof based on simple assumptions, essentially the concavity of the T-wave form. Moreover, it has the following advantages: (a) it is robust to measurement noise, since the computation in the algorithm mainly consists of an integration operation; (b) it is robust to wave form morphological variations and to baseline wander, since the consistency of the algorithm is essentially based on the assumption of T-wave concavity, and no threshold is needed to determine T-wave end location; (c) it is computationally very simple: the main computation is an integration operation over a sliding window and can be implemented as a simple finite impulse response (FIR) filter.

When evaluated with manually annotated ECG signals of the QT database [5] available on the PhysioNet web site [6], the proposed algorithm outperforms the other known algorithms evaluated with the same database, according to the most recent available publications up to our knowledge.

The paper is organized as follows. The proposed algorithm is first presented for continuous time signals in Section 2, then for discrete time signals in Section 3. The algorithm performance is then evaluated in Section 4 with the PhysioNet QT database. Finally, some concluding remarks are drawn in Section 5.

2. Algorithm in continuous time

For the convenience of presentation, the ECG signal is considered in this section as a continuous function of the time t, denoted by s(t). Based on QRS-complex detection (this task can be accomplished with any well established QRS detection method; see, e.g., [1]), for each cardiac cycle, a interval [t₁,t₂] is roughly delimited so that the T-wave end is inside this interval, with no overlap with the other wave forms (QRS and P). The following presentation assumes that such an interval is already chosen for each cardiac cycle.

In this paper, the time instants corresponding to the beginning and to the end of a T-wave are respectively denoted by t₁ and t₂. The T-wave length, denoted by L = t₂ - t₁, is generally an unknown value.

The proposed algorithm mainly consists of the computation of an indicator Λ(t) which reaches its maximum value when t = t₂. It is computed mainly through an integration operation in a sliding window. The window size W should be chosen such that 0 < W < L. It is possible to choose such a value with some rough knowledge on L. At each
time instant $t$, define the indicator

$$ A(t) = \int_{t-W}^{t} [s(\tau) - s(t)] d\tau $$

which can be understood as the area in the interval $[t-W, t]$ under the signal $s(t)$, but above the horizontal line crossing the point $(t, s(t))$, as illustrated in Figure 1(a).

Let us introduce some other useful notations

$$ t_{\text{top}} = \arg \max_{t \in [t_2,t_3]} s(t) $$

$$ h(t) = s(t-W) - s(t) $$

$$ h_{\text{max}}(t_2) = \max_{t \in [t-W,t_2]} |s(t) - s(t_2)| $$

and make the following assumptions.

Assumption 1: A T-wave is a differentiable concave function of the time in the interval $[t_1, t_2]$. It is followed by a straight line in the interval $[t_2, t_3]$ (a piece of the baseline, not necessarily horizontal).

Remark 1: Despite the approximative nature of this assumption, the resulting algorithm proposed in this paper produces accurate results when evaluated with the PhysioNet QT database, as shown in Section 4.

Assumption 2: If the straight-line segment of $s(t)$ in the interval $[t_2, t_3]$ has a negative slope $K$, then it is weak enough such that $|K| \leq h(t)/W$ for any $t \in [t_2,t_3]$.

Assumption 3: The segment of the signal in the interval $[t_2-W, t_1]$ preceding the T-wave is not a straight-line, but is upper bounded such that

$$ s(t) \leq s(t_{\text{top}}) $$

and satisfies the Lipschitz condition

$$ |s(t) - s(\tau)| \leq \frac{h_{\text{max}}(t_2)}{W}|t - \tau| $$

for any $t, \tau \in [t_2-W, t_1]$.

Now the property of the indicator $A(t)$ can be stated.

Proposition 1: Under Assumptions 1, 2 and 3, the indicator $A(t)$ defined in (1) satisfies the property

$$ t_2 = \arg \max_{t \in [t_1,t_3]} A(t) $$

A detailed proof of this property is presented in [7] and omitted here due to space limitation. Nevertheless, let us give an intuitive explanation of this property. Consider the particular case illustrated in Figure 1. From the position $t = t_2$ as illustrated in Figure 1(b), when the window is moved to the right side, part of the window is covering the baseline instead of the T-wave, then $A(t)$ decreases. When the window is moved to the left side, the bottom line of the area moves up with the value of $s(t)$, then the area $A(t)$ also decreases. Therefore, intuitively, in the illustrated particular case the T-wave end can be located by looking for the time $t$ maximizing $A(t)$. See [7] for a more rigorous proof covering different forms of the ECG signal segments proceeding and following T-waves.

3. Algorithm in discrete time

In practice, ECG signals are typically sampled at discrete time instants before being processed with a digital computer. Assume that the signal is sampled at a constant period $\Delta$, and with $s_k = s(k\Delta)$ being the $k$-th sampled signal value. Then the ECG signal becomes a sequence of values $s_k$ with $k = 1, 2, \ldots$. Accordingly, the definition of $A(t)$ is reformulated in discrete time as

$$ A_k = \sum_{j=k-w+1}^{k} (s_j - s_k) $$

where $w$ is the sliding window size in discrete time. In order to reduce the effect of measurement noise, in the above formula it is better to replace $s_k$ by $\bar{s}_k$, the mean value of the signal in a small window around $k$.

Some other notations already introduced in continuous time are also translated into discrete time, as summarized in Table 1.

A QRS-complex detection algorithm is first applied to detect R-peaks, with $R_i$ denoting the $i$-th detected R-peak (discrete time) instant. Inside each detected RR interval, two discrete time instants $k_a$ and $k_b$ are chosen to confine the T-wave end search. Then for each instant $k$ between $k_a$ and $k_b$, the value of $A_k$ is computed and the T-wave end is located at the value of $k$ maximizing $A_k$.

Up to now in this paper, only positive T-waves have been considered. Following [3], T-wave morphologies can be classified as positive, negative, biphasic (positive-negative or negative-positive), ascending only, and descending only.

For negative T-waves, the same algorithm can be applied to $-s(t)$ instead of $s(t)$, or equivalently, the minimum of $A(t)$ is searched for instead of its maximum. The other morphologies can be dealt with similarly.

<table>
<thead>
<tr>
<th>Continuous time</th>
<th>Discrete time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$</td>
<td>$A_k$</td>
</tr>
<tr>
<td>$A(t)$</td>
<td>$w$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>$k_a$</td>
</tr>
<tr>
<td>$t_4$</td>
<td>$k_b$</td>
</tr>
<tr>
<td>$t_5$</td>
<td>$k_2$</td>
</tr>
</tbody>
</table>

Table 1. Notation correspondence between continuous time and discrete time.
Since the T-wave morphology is a priori not known, the following simple rule is adopted. Search for the instants $k'$ and $k''$ respectively maximizing and minimizing $A_k$. If $A_{k'}$ and $A_{k''}$ have comparable absolute values (decided based on a threshold value), then the T-wave is considered as biphasic, and the latest of the two instants $k'$ and $k''$ is taken as the location of the T-wave end. Otherwise, the largest absolute value $|A_{k'}|$ or $|A_{k''}|$ corresponds to the T-wave end.

The complete algorithm in discrete time is summarized in the following.

Algorithm summary.
1. Choose the sliding window size $w$ and the smoothing window size $p << w$. Choose also a threshold $\lambda > 1$ for T-wave morphology classification.
2. Read two successively detected R-peak instants $R_i$ and $R_{i+1}$.
3. Choose the values of $k_a, k_b$ between $R_i$ and $R_{i+1}$ to confine the T-wave end search.
4. For each instant $k = k_a, k_a + 1, \ldots, k_b$, compute
   \[
   s_k = \frac{1}{2p + 1} \sum_{j=k-w}^{k+p} s_j
   \]
   \[
   A_k = \sum_{j=k-w+1}^{k+p} (s_j - s_k)
   \]
5. Find
   \[
   k' = \arg \max_{k_a \leq k \leq k_b} A_k
   \]
   \[
   k'' = \arg \min_{k_a \leq k \leq k_b} A_k
   \]
6. If
   \[
   \frac{1}{\lambda} < \frac{|A_{k'}|}{|A_{k''}|} < \lambda
   \]
   then the T-wave end in the current RR interval is located at
   \[
   k_2 = \max(k', k'')
   \]
   otherwise,
   \[
   k_2 = \arg \max_{k \in \{k', k''\}} |A_k|
   \]
7. Increase $i$ by one and go back to step 2. □

Remark 2: The main part of this algorithm, the computation of $A_k$, is in fact a linear combination of the signal values from $s_{k-w+1}$ to $s_{k+p}$. It can thus be simply implemented as a finite impulse response (FIR) filter. □

The choice of $k_a, k_b$, in each RR interval has an important influence on the result of T-wave end detection. They should define an interval large enough to contain the T-wave end, and small enough in order to avoid overlap with the other wave forms. As an example, the rule for choosing $k_a, k_b$ when the algorithm is applied to the PhysioNet QT database will be given in the next section.

4. Evaluation with the PhysioNet QT database

The PhysioNet QT database [5] has been built by researchers to serve as a reference for the validation and comparison of ECG processing algorithms. The signals in this database have been manually annotated by cardiologist experts for various events. Only the T-wave end annotations are used in this paper. In 105 records, a total of 3542 T-wave ends have been annotated by one cardiologist, and 402 T-wave ends in 11 records have been annotated by another cardiologist. Each record consists of two leads of 15 minutes ECG signals sampled at 250 Hz.

The signals are first filtered to reduce baseline wander before the application of the proposed detection algorithm. For simplicity, the Fast Fourier Transform (FFT) of each signal is first computed and then the frequency components below 0.5 Hz are truncated before the inverse FFT is computed.

The following parameters have been used for the processing of the QT database which is sampled at 250 Hz.

The sliding window $w = 32$, the smoothing window $p = 4$, the threshold for T-wave morphology test $\lambda = 6$. Denote $RR_i = R_{i+1} - R_i$, the search interval is chosen as

\[
\begin{align*}
  k_a &= \begin{cases} 
  R_i + [0.15RR_i] + 37 & \text{if } RR_i < 220 \\
  R_i + 70 & \text{if } RR_i \geq 220
  \end{cases} \\
  k_b &= \begin{cases} 
  R_i + [0.7RR_i] - 9 & \text{if } RR_i < 220 \\
  R_i + [0.2RR_i] + 101 & \text{if } RR_i \geq 220
  \end{cases}
\end{align*}
\]

where $[x]$ means the largest integer smaller than or equal to $x$, and $\lfloor x \rfloor$ means the smallest integer greater than or equal to $x$.

In order to evaluate the accuracy of the proposed algorithm, the mean and the standard deviation (STD) of the location errors (the difference between the automatically detected T-wave ends and the manually annotated T-wave ends) are computed.

Up to our knowledge, the results of 4 different detection algorithms evaluated with the PhysioNet QT database have been published [8], [9], [3] and [4]. For the purpose of comparison, the results of the first 3 algorithms are recalled in Table 2. It has not been possible to compare with the results of [4], because only 3000 annotated T-wave ends (out of 3542 available) were used for the results reported in this publication without indicating how these 3000 annotations were selected, and the mean value of the detection errors is not reported.

In Table 2, the first row (of numerical values) is the results for all the signals annotated by cardiologist 1, the other two rows are the results for the 11 records annotated by both cardiologists. Column 1 is the reference annotation used for error computation, column 2 is the number of records, column 3 is the number of annotated T-wave
<table>
<thead>
<tr>
<th>Reference</th>
<th>num. of records</th>
<th>num. of annot.</th>
<th>this paper mean</th>
<th>STD</th>
<th>WT mean</th>
<th>STD</th>
<th>LPD mean</th>
<th>STD</th>
<th>TU mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardio. 1</td>
<td>105</td>
<td>3542</td>
<td>0.31</td>
<td>17.43</td>
<td>-1.6</td>
<td>18.1</td>
<td>13.5</td>
<td>27.0</td>
<td>0.8</td>
<td>30.3</td>
</tr>
<tr>
<td>Cardio. 2</td>
<td>11</td>
<td>487</td>
<td>-7.47</td>
<td>17.18</td>
<td>-9.7</td>
<td>18.1</td>
<td>-10.8</td>
<td>20.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. Evaluation of algorithms on the QT database. The first row (of numerical values) is the results for all the signals annotated by cardiologist 1, the other two rows are the results for the 11 records annotated by both cardiologists. Column 1 is the reference annotation used for error computation, column 2 is the number of records, column 3 is the number of annotated T-wave ends, column 4 is the results (mean and standard deviation (STD) of detection errors) of the proposed method, the following columns are the results of the algorithms WT [3], LPD [8] and TU [9]. The time unit is millisecond. The missing values are not reported in the original publication.

The computation of the mean and the standard deviation of errors presented in Table 2 follows the method of [3]. Some variant evaluation methods and the corresponding numerical results are presented in [7].

5. Conclusion

A new algorithm for T-wave end location has been presented based on an indicator signal with mathematically proved consistency. It is robust to measurement noise, since its computation mainly consists of an integration operation. It is robust to wave form morphological variations and to baseline wander, since the consistency of the algorithm is essentially based on the assumption of T-wave concavity, and the location of T-wave does not require any threshold. The computation burden of the algorithm is very low: its main computation can be implemented as a simple FIR filter.

Satisfactory results have been obtained when this new algorithm is evaluated on the PhysioNet QT database. In terms of error mean value and standard deviation, it outperforms the other algorithms evaluated on the same database, according to recent publications.

By examining the results of the algorithm, it has been observed that large errors are mainly caused by incorrect morphological classification. In order to further improve the results, it is important to develop more accurate and robust methods for morphological classification.

References


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